Quasi-optical design and Focal Plane Array for THz and millimeter wave imaging systems

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Layout of the talk

1. Introduction.
2. Design parameters and considerations for imaging systems.
3. Imaging ability of focusing systems.
4. Quasi-optical Design approaches for imaging systems.
5. Description of a specific systems and results.
6. DSP and Super-Resolution (SR).
Introduction
Definitions

What is imaging??

• Imaging can be considered to be the process of measuring the radiation arriving from different directions.
• Imaging is approximation.

What is Quasi-Optics??

Quasi-Optics deals with the propagation of a beam of radiation that is reasonably well collimated but has relatively small dimensions which measured in wavelength, transverse to the axis of propagation.
How to obtain an image of a scene (1)

1) Scanning a single pixel receiver
How to obtain an image of a scene (2)

2) Focal Plane Array (FPA)

Imaging mirror

Collimated beam

100 GHz source

OPM

Object

Diagonal mirror

FPA
How to obtain an image of a scene (3)

3) Interferometric imaging
Passive and active imaging

Why passive imaging system?

• All Natural objects whose temperatures are above absolute zero emit passive millimeter-wave radiation.
• Non invasive
• Requires very sensitive and expansive detector and electronics
• Usually based on scanning imaging system
• Limited dynamic range
• Defused scattering from surface.

Why Active imaging system?

• Requires illumination of the object.
• Simpler electronics and better SNR compare to passive
• Can be based on direct or heterodyne detection
• Can be based on FPA, scanning or interferometric imaging system
• Large dynamic range
• Specular reflections
Advantages of MMW and THz imaging

• Millimeter wavelength propagation is superior to that found in the IR and VIS
• Millimeter wavelengths clearly offer better angular resolution
• Remote sensing of trace gases.
Design parameters and considerations for imaging systems
Sensitivity (1)

Passive imaging systems

• Very low emission from the target in THz and MMW compare to the background and to IR band

• Minimum detectable temperature difference is given by:

\[ \Delta T_{RMS} = \frac{N_T}{\sqrt{B \tau}} \]

Where \( N_T \) is the noise temperature of the imager, \( B \) is the RF bandwidth and \( \tau \) is the post detector time integration time of the imager.

• For the highest thermal sensitivity, \( N_T \) must be low and \( B \) and \( \tau \) must be big as possible. For example a typical system \( N_T \) may be 3000 K, \( B \) may be 3 GHz and \( \tau 10 \) msec. This gives \( T_{RMS}=0.5 \) K.
Sensitivity (2)

Active imaging systems

• Depend on the imaging configuration and illumination source.
• The minimal detected power is given by:

\[ \Delta P_{RMS} = kT_{RMS}B \]

Where \( k \) is Boltzmann constant and \( B \) is the electronic bandwidth.

• If the signal is sufficiently strong, it can be directly detected without any RF processing.
• The noise is set by the noise fluctuations in the detector element rather then pre-detection bandwidth which is generally not a critical parameter.
Focal Plane Array’s design (1)

**Diffraction limited system** - is a system in which, the image is limited by the wave nature of radiation.

**Airy disk**

The diameter of the central disk is:

\[ B_{diff} = 2.44\lambda(F/\#) \]

In 100 GHz and \( F/\# = 2 \) the diffraction bluer is about 7.5mm

The trend to decrease the pixel sizes in focal plane arrays to increase the system's resolution requires awareness of this limit set by nature.
Focal Plane Array’s design (2)

According to Nyquist sampling theory, spatial resolution is one half of focal spot which is decided by operation frequency, aperture of the optical equipment and focal length. Thus the spacing between pixels should be:

\[ \Delta x = \frac{1}{2} \cdot \frac{F\lambda}{D} \cdot 2.44 \]

In \( f/D=1 \) systems, we require that the spacing between two pixels will be not much than about \( \lambda/2 \).
Field of view

The field of view (FOV) is the extent of the observable world that is seen at any given moment by human or imaging systems.
Imaging ability of focusing systems
Performance and requirements (1)

The critical parameters are:

1) Antenna gain,
2) Beam size
3) Beam quality over the range of angles scanned
Performance and requirements (2)

Difficulties:

1. Focal plane arrays with $10^4$ pixels are being seriously considered, with the result that scan angles x100 beam widths off boresight must be considered.

2. System aperture diameters are relatively small, typically only a few hundred wavelengths for commercial systems, and often less. Blockage loss for symmetric reflector systems of such limited size is generally excessive.

3. The limited volume available restricts f/D for lens systems to values $\leq 1.25$, which seriously restricts imaging capability. For this same reason, off-axis optical systems are generally not acceptable.
Quasi Optical Design approaches for imaging systems
Design methods of imaging system (1)

1) Geometrical optics (ray tracing)

Ray tracing is based on three optical principles:

1) Conservation of energy along the ray tube
2) Snell’s law at boundaries
3) Electrical path length and Fermat principle (principle of least time)

Ray tracing is used to design systems because it is simple and effective especially in VIS and IR where large and moderate $F/# (>10)$ optics is used. It is not suitable for small $F/# (\approx 1)$ and thick lenses which are very common in THz and MMW quasi-optics.
Design methods of imaging system (2)

2) Gaussian beam method

This method is based on the Gaussian solution of the paraxial wave equation

\[
\left( \nabla^2 + k^2 \right) \psi = 0
\]

Where \( \psi \) is single component of electromagnetic wave. Letting the wave propagate in z direction, we can write any electric distribution as:

\[
E(x, y, z) = u(x, y, z) \exp(-jkz)
\]

Where \( u \) is a complex scalar function that defines the non-plane wave part of the beam
Design methods of imaging system (3)

2) Gaussian beam method

In the paraxial approximation the solution for the normalized fundamental Gaussian electric field distribution as function of distance from beam axis $r$ and location $z$ is obtained:

$$E(r, z) = \left(\frac{2}{\pi \omega^2}\right)^{0.5} \exp \left[ \frac{-r^2}{\omega^2} - jkz - \frac{j\pi r^2}{\lambda R} + j\phi_0 \right]$$

Where $\omega$ and $R$ are functions of $z$ and stand for the Gaussian beam size and the radius of curvature respectively. $\phi_0$ is the Gaussian beam phase shift and it is also function of $z$. 
Design methods of imaging system (4)

2) Gaussian beam method

Gaussian Beams can imitate quite good the propagation of MMW and THz. Gaussian beam method has good performances on designing the quasi-optical subsystem if the truncation effect is considered, but still a bit different from the actual situation due to phase shift of wave fronts on the boundaries of quasi-optical components.
Design methods of imaging system (5)

3) ABCD matrix and Gaussian beam parameter $q$

The Gaussian beam parameter $q(z)$ is obtained by the solution of the paraxial wave equation:

$$\frac{1}{q(z)} = \frac{1}{R(z)} - \frac{j \lambda}{\pi(\omega(z))^2}$$

Where we define the Gaussian beam as:

$$u(r, z) = A(z) \exp \left[ -\frac{jkr^2}{2q(z)} \right]$$

It can be shown that the ABCD method used for Geometrical optic can be applied to the complex Gaussian beam:

$$r_{out} = Ar_{in} + Br'_{in}$$
$$r'_{out} = Cr_{in} + Dr'_{in}$$

$$\begin{bmatrix} r_{out} \\ r'_{out} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} r_{in} \\ r'_{in} \end{bmatrix}$$
Design methods of imaging system (6)

3) ABCD matrix and Gaussian beam parameter $q$

Since the radius of curvature is defined by $R=r/r'$, we can combine the two parts of the above equations into an expression for the radius of curvature:

$$R_{out} = \frac{A \cdot R_{in} + B}{C \cdot R_{in} + D}$$

$\downarrow$

$$q_{out} = \frac{A \cdot q_{in} + B}{C \cdot q_{in} + D}$$

The ABCD law is an enormous aid to quasi-optical analysis, since all the geometrical optical ray theory can be applied to Gaussian beam representation of a system.
Design methods of imaging system (7)

3) ABCD matrix and Gaussian beam parameter $q$

The phase change in thick lens and truncation effect are not fully considered in this method, so it has been proved to be not accurate and can not explain the measurements.

4) Hybrid approach

The design is divided into three parts:
1) Gaussian beam propagation between feed elements and optical component.
2) Ray tracing through the optical component.
3) A single diffraction calculation of propagation to the far-field or a specified plane where the properties of the beam can be examined.
Quasi optical components (1)

1) Dielectric lens design

According to Fermat’s principle of least time:

\[ |SP| + n|PP'| = |SV| + n|VP''| \]

\[ r = \frac{(n - 1)F}{n \cos \theta - 1} \]

Given that lens has \( n > 1 \), this define a hyperbola

The distance \( r_h \) is given by:

\[ r_h^2 = 2Fz(n - 1) + z^2(n^2 - 1) \]
Quasi optical components (2)

1) Dielectric lens design

Spherical–plano lens does not satisfy Fermat's principle for appreciable off axis distance or angles.
2) Reflective focusing (mirrors) design

There are two commonly used forms for reflective focusing: Ellipsoid and Paraboloid.

**Paraboloid.**

\[
z = \frac{x^2 + y^2}{4F_p}
\]

In spherical polar coordinates.

\[
\rho = \frac{2F_p}{1 + \cos \theta}
\]

Where the origin is in the focal point.
Quasi optical components (4)

2) Reflective focusing (mirrors) design

We will consider $\phi = 0$, thus we have the $xz$ plane.

An incident ray parallel to $z$ axis makes an angle $\theta_i$ relative to local normal $z'$. According to Snell’s law, this is also the reflected angle. The reflected ray will pass through the focal point.

$$\theta = 2\theta_i \quad \rightarrow \quad x = 2 \sin \theta_i \cos \theta_i \quad \rightarrow \quad \rho = \frac{F_p}{\cos^2 \theta_i}$$
Quasi optical components (5)

2) Reflective focusing (mirrors) design

• On-axis paraboloid is not very useful due to partialiy blockage of the incident beam.
• Off-axis paraboloid are generally employed.

Properties of using reflective quasi optical components

1. Freedom from absorption and reflective losses
2. High power handling capabilities
3. Enhance the performance of large apertures antennas using special surfaces-primarily “canonical” paraboloidal, ellipsoidal
4. We should take into account skin depth and metal resistivity
5. Off-axis elements do, in general, introduce both beam distortions and cross-polarization.
Quasi optical components (6)

2) Reflective focusing (mirrors) design

Metal reflection

For imperfect conductors, the critical quantity is the equivalent transmission line impedance given by:

$$Z_m = (1 + j)(\pi f \mu_0 \rho)^{0.5}$$

The skin depth is given by:

$$\delta = \sqrt{\frac{\rho}{\pi f \mu_0}}$$

Where $\rho$ is the bulk DC resistivity

<table>
<thead>
<tr>
<th>Material</th>
<th>Skin depth (µm) @ 1THz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>0.08</td>
</tr>
<tr>
<td>Copper</td>
<td>0.07</td>
</tr>
<tr>
<td>Gold</td>
<td>0.08</td>
</tr>
<tr>
<td>Nickel</td>
<td>0.15</td>
</tr>
<tr>
<td>Titanium</td>
<td>0.37</td>
</tr>
<tr>
<td>Silicon (un-doped)</td>
<td>12,732</td>
</tr>
<tr>
<td>Germanium (un-doped)</td>
<td>341</td>
</tr>
</tbody>
</table>

Table 3: Skin depth for materials @ 1THz
Quasi optical components (7)

2) Reflective focusing (mirrors) design
Quasi optical components (8)

2) Reflective focusing (mirrors) design

**Distortion** (off-axis reflective component)
Quasi optical components (9)

2) Reflective focusing (mirrors) design

Distortion (off-axis reflective component)
Quasi optical components (10)

2) Reflective focusing (mirrors) design

Cross-polarization (off-axis reflective component)

In general, the curvature of the surface of a reflective focusing element and the resultant change in the direction of the local surface normal will produce a change in the direction of polarization of a linearly polarized beam

Surface accuracy

Specular Reflection

Diffuse Reflection
Design methodology and general guidelines

System performance specifications

Choice of quasi-optical components and system architecture

Critical beam waist radii

Quasi-Optical configurations

Beam truncation

Beam waist location

Beam coupling and frequency dependence

Evaluation and Optimization

Final system configuration
Design methodology and general guidelines

1) Architecture and components

- **System Architecture** - Basic arrangement of quasi-optical components
- **Components** – mirrors, lenses, polarizers, frequency selective surfaces, diffraction gratings, Fabry-Perot Interferometer, resonators and feed horns.

**Frequency selective application**
Resonators and interferometers

Fabry-Perot resonator for gas attenuation measurements
Design methodology and general guidelines

2) Beam waist radii

Beam waist criticality of various quasioptical components

<table>
<thead>
<tr>
<th>Noncritical</th>
<th>Polarization grid, Absorption load</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moderately critical</td>
<td>Perforated plate filter, Diffraction grating, Frequency selective surface, Dielectric filled Fabry-perot interferometer</td>
</tr>
<tr>
<td>Highly critical</td>
<td>Dual-beam interferometer, Fabry-perot interferometer</td>
</tr>
<tr>
<td>Determined by component</td>
<td>Resonator, Feed horn</td>
</tr>
</tbody>
</table>

For example, a dielectric filled FPI is not necessarily less critical in terms of beam radius than any air filled unit, but it will be less sensitive since:

\[
D = \frac{c}{\Delta \nu z c (1 - |r_2|^2) n^2}
\]
Design methodology and general guidelines

3) Beam waist location
The beam waist radii is important, but also the location of the beam waist is required for efficient coupling. The variation in beam waist location as a function of operating system frequency is one of major limitation.

For a focusing element using ABCD method (thin lens):

\[
M = \begin{bmatrix} 1 & d_2 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1/f \end{bmatrix} \cdot \begin{bmatrix} 1 & d_{in} \\ 0 & 1 \end{bmatrix}
\]

\[
= \begin{bmatrix} 1 - \frac{d_2}{f} & d_{in} + d_2 \left(1 - \frac{d_{in}}{f}\right) \\ -1/f & 1 - \frac{d_{in}}{f} \end{bmatrix}
\]
To find the location and size of the beam waist in region 2, we can use the general ABCD matrix with the Gaussian parameter \( q \):

\[
q_{out} = \frac{(A+Cd_{out}) \cdot jz_c + [(A+Cd_{out})d_{in} + (B+Dd_{out})]}{C \cdot jz_c + Cd_{in} + D}
\]

Substituting A, B, C and D and impose imaginary \( q (R \to \infty) \) we obtain :

\[
q_{out} = \frac{\left(1 - \frac{d_2}{f}\right) \cdot jz_c + d_{in} + d_2 \left(1 - \frac{d_{in}}{f}\right)}{\left(1 - \frac{d_{in}}{f}\right) - jz_c/f}
\]

\[
\frac{d_{out}}{f} = 1 + \frac{d_{in}/f - 1}{\left(d_{in}/f - 1\right)^2 + z_c^2/f^2}
\]

\[
\omega_{0out} = \omega_{0in} \left[\left(\frac{d_{in}/f - 1}{f^2} + z_c^2/f^2\right)\right]^{0.5}
\]
Design methodology and general guidelines

4) Quasi-Optical configuration

- Ordering and arrangement of components
- Reflective or Refractive optics
- Performance and optimization

Parameters for optimum coupling of various feed structure

| Feed type                  | $\omega/\alpha$ | $|c_o|^2$ |
|----------------------------|-----------------|-----------|
| Corrugated circular        | 0.64            | 0.98      |
| Corrugated square          | 0.35            | 0.98      |
| Smooth walled circular     | 0.76            | 0.91      |
| Dual mode                  | 0.59            | 0.98      |
| Rectangular                | 0.35            | 0.88      |
| diagonal                   | 0.43            | 0.93      |
5) Beam truncation

- One of the greatest practical issues concerned with quasioptical system design.
- The fraction of power lost at radius greater than $r_e$ is defined to be $F_{lost}(r_e)$.
- $F(r_e)$ is the fraction within radius $r_e$, so that $F_{lost}(r_e) = 1 - F(r_e) = T_e(r_e)$.

$$T_e = \frac{P(r_e)}{P(0)} \implies T_e(r_e) = \exp\left(\frac{-2r_e^2}{\omega^2}\right)$$

And:

$$F_e(r_e) = \int_{0}^{r_e} |E(r)|^2 \cdot 2\pi r dr = 1 - T_e(r_e)$$
In addition to power loss effect, there are two more effects:

1) Broadened of the Gaussian beam (reduction of original $\omega_0$).

For moderate truncation levels $T_e(dB) \leq 20dB$:

$$\frac{\omega_{0\text{eff}}}{\omega_0} = \frac{0.40\sqrt{T_e(dB)}}{1.6 + 0.021T_e(dB)}$$

For $T_e(dB)=10dB$ we will have 0.7 times smaller waist

For low truncation levels $T_e(dB) \geq 20dB$:

$$\frac{\omega_{0\text{eff}}}{\omega_0} = 1 - \sqrt{T_e}$$

For $T_e(dB)=30dB$ we will have about 0.03 times smaller waist
2) Sidelobes in the far-field radiated pattern.
7) Coupling, Frequency dependence And Optimization

The first step is to employ the ABCD matrix with $q$. It is effective to start with a waist produced by coupling device – feed horn (INPUT). We have to check the OUTPUT.

In addition there are three important parameters not directly connected to Gaussian beam propagation model: Loss, Polarization behavior and frequency response. For imaging system it is recommended to check the diffraction from the imaging component to the FPA.
System design examples and results
System design examples and results

1) 4 off-axis parabolic mirrors (focal) spectrometer
System design examples and results

2) 4 off-axis parabolic (15°) mirrors (focal) spectrometer
System design examples and results

3) T/R system
System design examples and results

3) 2 off-axis parabolic mirrors MMW spectrometer transmission measurements
1. Polyethylene (HDPE polyethylene)
2. Pro-opiomelanocortin (POMC)
3. Poly(methyl methacrylate)(PMMA)
4. Polycarbonates (PC)
5. Polypropylene (PP)
System design examples and results

4) 2 off-axis parabolic mirrors MMW spectrometer reflection measurements
System design examples and results

5) Design of 8X8 GDD Focal Plane Array
System design examples and results
5) Design of 2x36 column GDD Focal Plane Array
6) Design of MMW imaging system
System design examples and results

7) Simulation results for metal “F” shape object
System design examples and results
DSP and super-resolution
Super resolution methods for MMW imaging