

# Directed Information Optimization and Capacity of the POST Channel with and without Feedback

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## ABSTRACT

We will first review the important roles of directed information and causal conditioning in problems with causal constraints, such as communication, estimation, gambling, statistical inference and more. Then we will talk about the convexity properties of directed information. Using these properties, we develop a numerical algorithm for optimizing any finite-letter expression of the directed information. These finite-letter expressions can be used, for instance, to find upper and lower bounds on the capacity of any finite state channel with feedback.

In the second part of the talk, we will consider finite state channels where the state of the channel is its previous output. We refer to these as POST (Previous Output is the State) channels. We begin with a simple POST channel. This channel has binary inputs and outputs, where the state determines whether the channel behaves as a Z or an S channel. We show that the non-feedback capacity of the simple POST channel equals to its feedback capacity, despite the memory of the channel. The proof of this result is based on showing that the induced output distribution, when maximizing the directed information in the presence of feedback, can also be achieved by an input distribution that does not utilize the feedback. Finally, we will show that this surprising result extends to any binary POST channel where the previous output determines whether the current channel will be binary with parameters  $(a,b)$  or  $(b,a)$ .

The talk is largely based on joint work with Himanshu Asnani, Iddo Naiss and Tsachy Weissman.