Airplane boarding and space-time geometry.

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Ben-Gurion U

Disk scheduling problem

- You send the disk drive in your laptop or PC a bunch of I/O requests. Modern disk drives can reorder these requests.
  What is the optimal ordering?
  How long will it take to answer them?

Disk scheduling

- Classical problem, lots of research
- Andrews, Bender and Zhang (1996) gave an optimal algorithm when there is no acceleration-deceleration of the disk head.
- How long does it take the disk to execute the algorithm?

Disk scheduling problem

- Probabilistic approach – some files are more popular than others, should be taken into account.

A wise man once said:

“Disk scheduling is like airplane boarding “
(Steve Skiena)

So lets study airplane boarding!

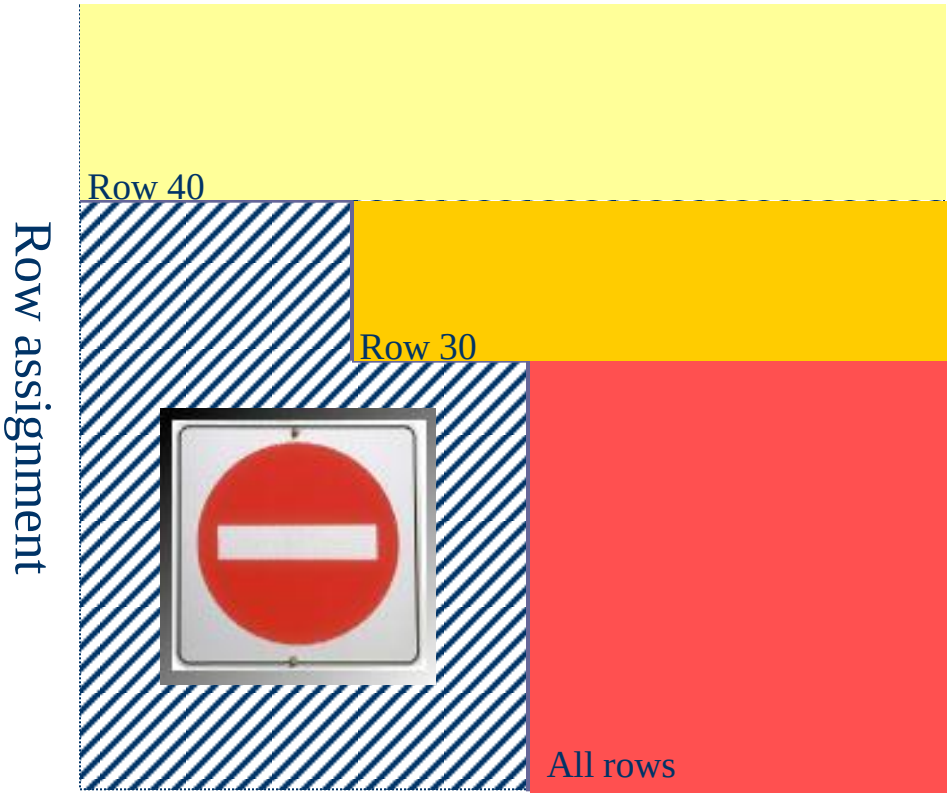
Airplane boarding: Description of problem

- Passengers line up to board the airplane
- Airline policy: “People from row 40 and above are welcome to board the plane”
- Airplane arrangement: Distance between rows, # of passengers/row.
- Find best policy given airplane arrangement.

How to analyze

- Passenger coordinates: queue position, row number.
- Airline policy=correlation between coordinates=non-uniform joint distribution.

Typical airline policy

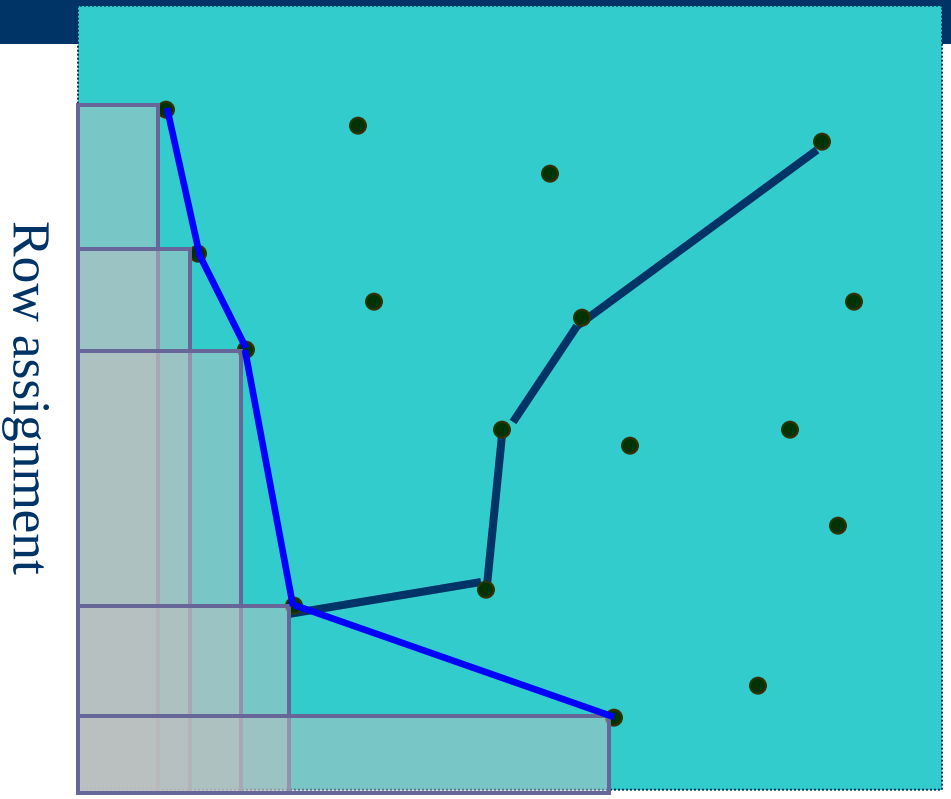


The supermodel

- Passengers are infinitely thin and can walk quickly one after the other along a narrow aisle.
- Spacious airplane, few seats per row, a lot of leg room.
- Not realistic.

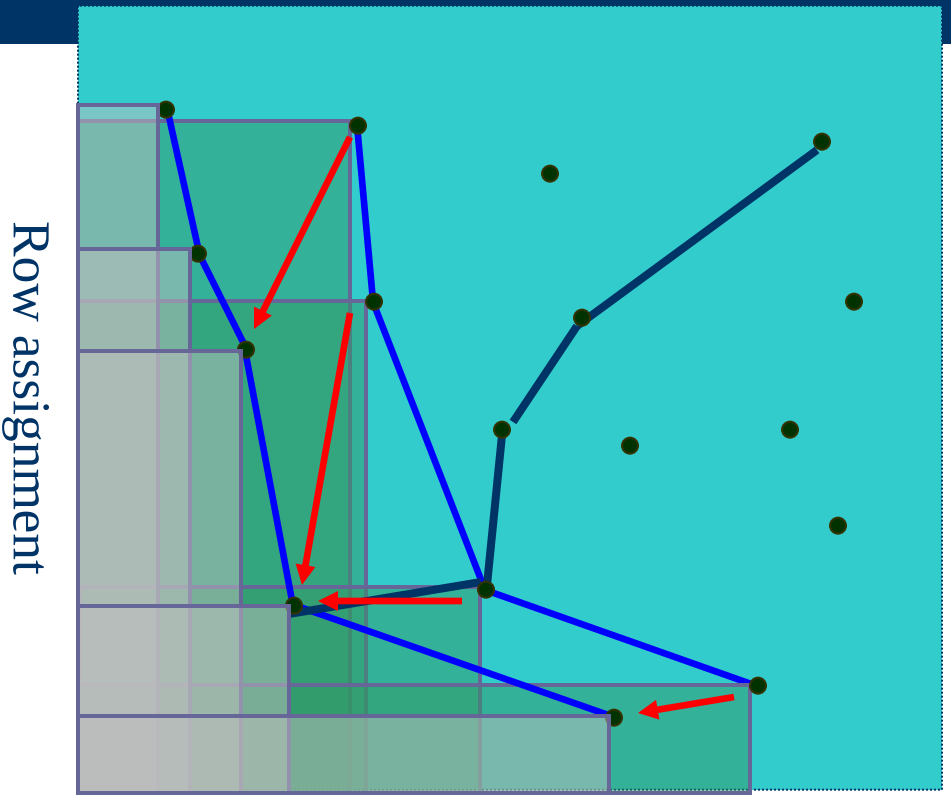
Think again!

Peeling process



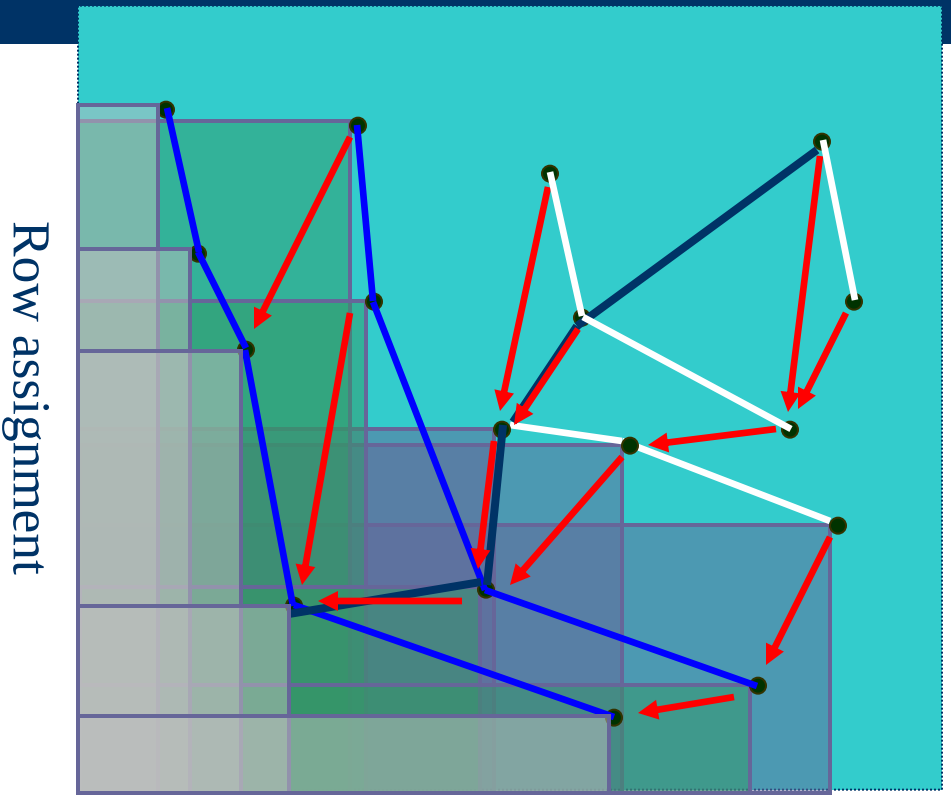
First
Round

Peeling process



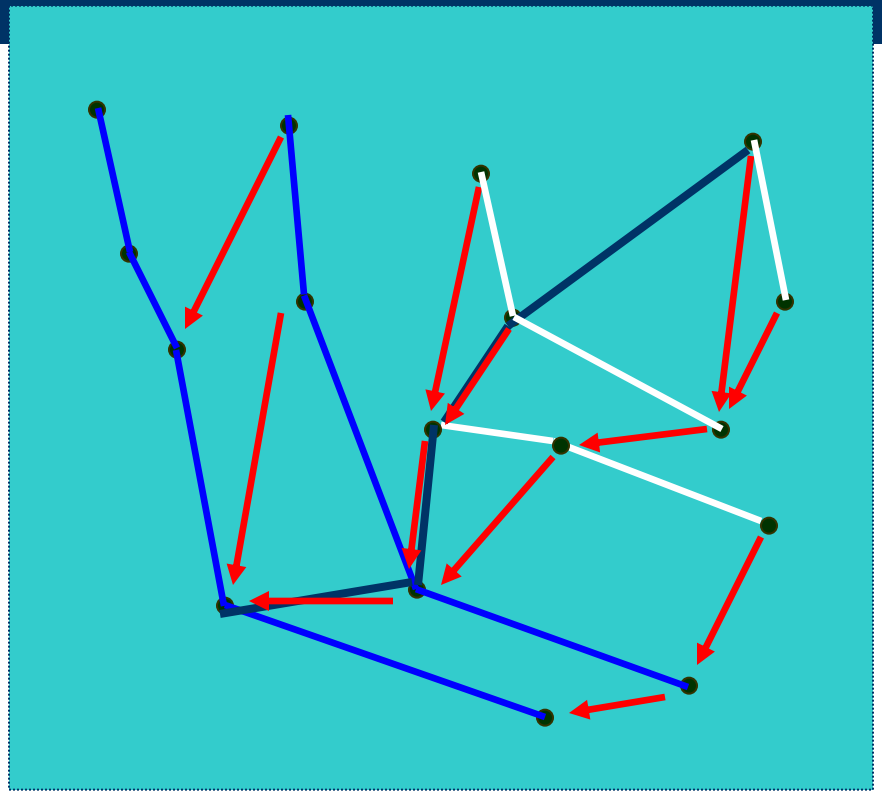
Second
Round

Peeling process



Third-Fifth Round

Peeling process

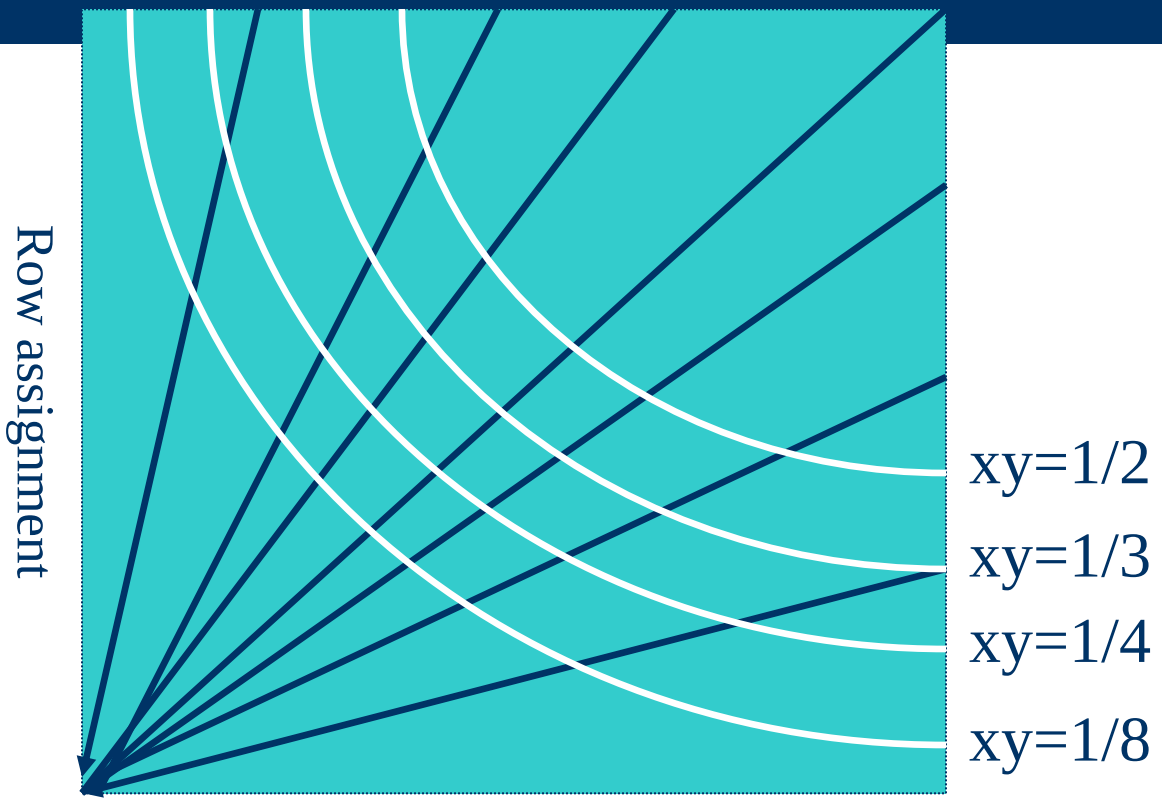


Row assignment

Queue joining time

End Result

End result with many points



General case

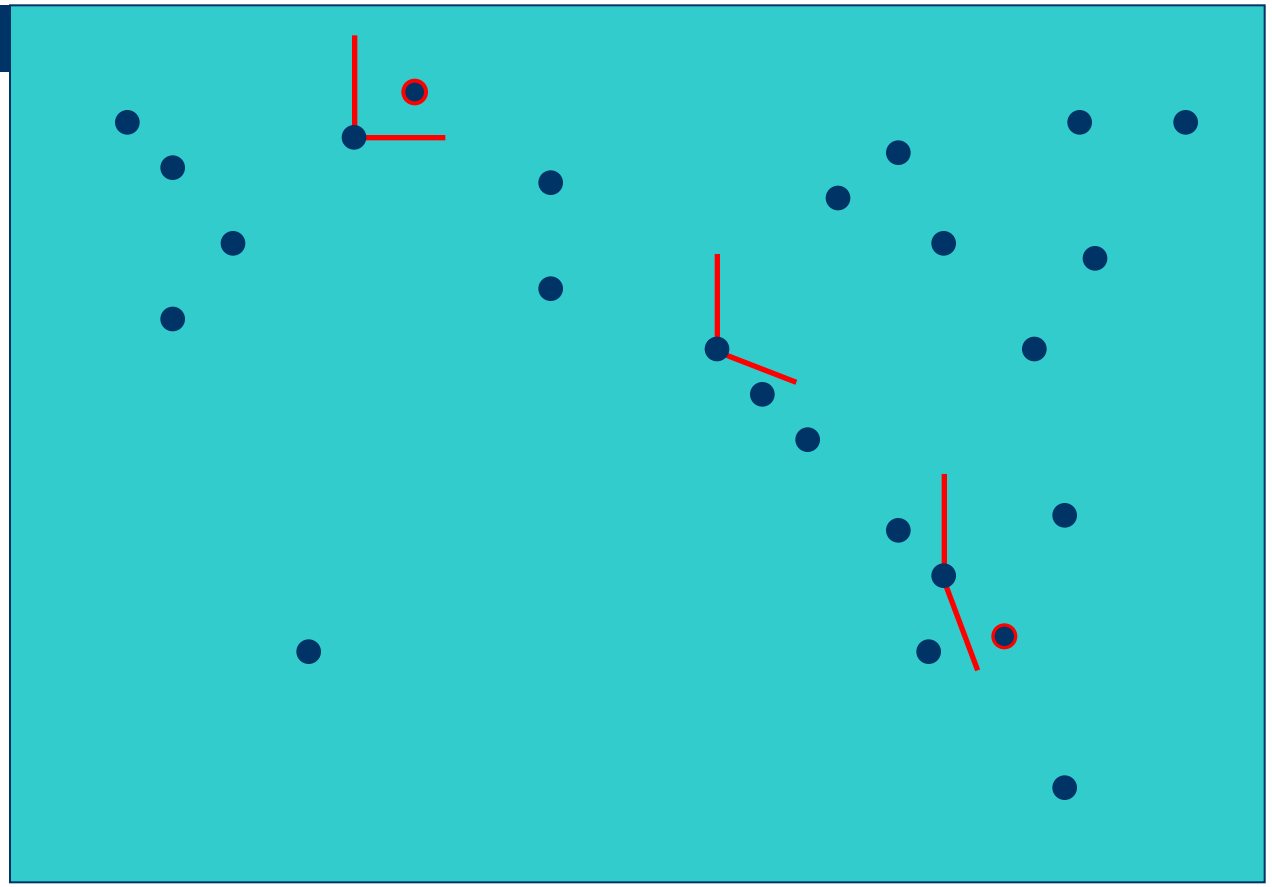
Results of computations

Back-to-front is good in the supermodel

Results of computations

- Back-to-front not so good for mere mortals. (with Elkin, Khachaturov)
- Random is good
- Outside-in, with a touch of back-to-front, probably better
- Can do better if given complete control over queue position, Van Landeghem and Beuselinck (VB) 2002, B. et. Al., Steffen 2008. Steffen thinks its realistic.

General case: Geometric representation



General case

- At each point, density + blocking cone, Hmmm… sounds familiar, well isn't that what space-time geometry is all about.
- Yes!!

The universe as a video game

- “Prizes” are scattered throughout space and time according to a fancy formula, Einstein’s field equations.
- When you “swallow” a prize you age.
- Restriction on maneuvering: Cant pass the speed of light.
- According to Einstein: During free fall you maximize the number of prizes you swallow.
- Trajectory is called a geodesic.
- How cool is that?

Interesting observation

- If you take a completely random path you also follow a geodesic!
- No optimization needed!
- Perhaps god plays dice?

In collaboration with

- Steve Skiena, Dani Berend, Luba Sapir, Natan stolyarov, Vassilii Khachaturov, Michael Elkin.
- Credit also goes to Deuschel and Zeitouni, the interesting observation is an extension of their result.

Some references

- Arxiv: gr-qc 0702140
- J. of Phys. A, L453-L459, 2006.
- B., Berend, Sapir, Skiena, Stolyarov, “Analysis of airplane boarding”, Hopefully to appear in Operations Research.
- Airplane boarding page by Menkes Van den Briel
- My web page
- Book on performance analysis of storage systems, AMS student series, 2010-2011?

Related math surveys

- P. Deift, ICM 2006 plenary talk, arxiv+video.
- R. Stanely, ICM 2006 plenary talk.
- Aldous and Diaconis, Patience sorting…, Bull. Of the AMS.

A message

- This ends the entertainment portion of our talk, on behalf of B. and his coauthors, we would like to thank you for joining our talk today. We know that as consumers of seminar talks you have many choices and we thank you for choosing us for your talk today, we hope to be able to serve you again soon and wish you a pleasant journey, whatever your final destination may be. If you are continuing with us to disk scheduling your gate will be the next slide, for connect-the dots the 2nd slide, for surface growth models 3rd slide, for maximal layers 4^{th} slide,…

Disk scheduling as a wave front. Time traveling on a daily basis!