Clustered Local Decoding and Cognitive Users in Wyner-Type Cellular Models

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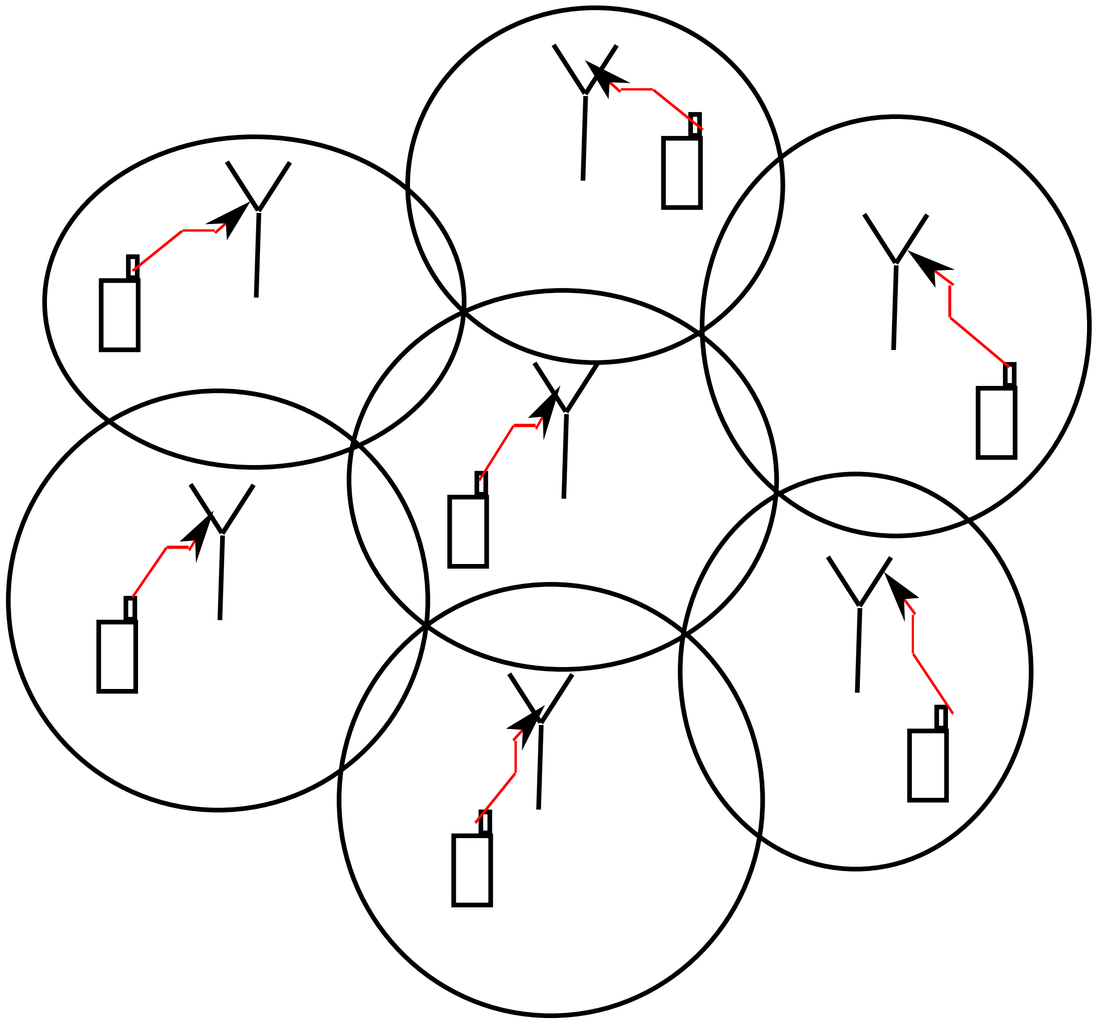
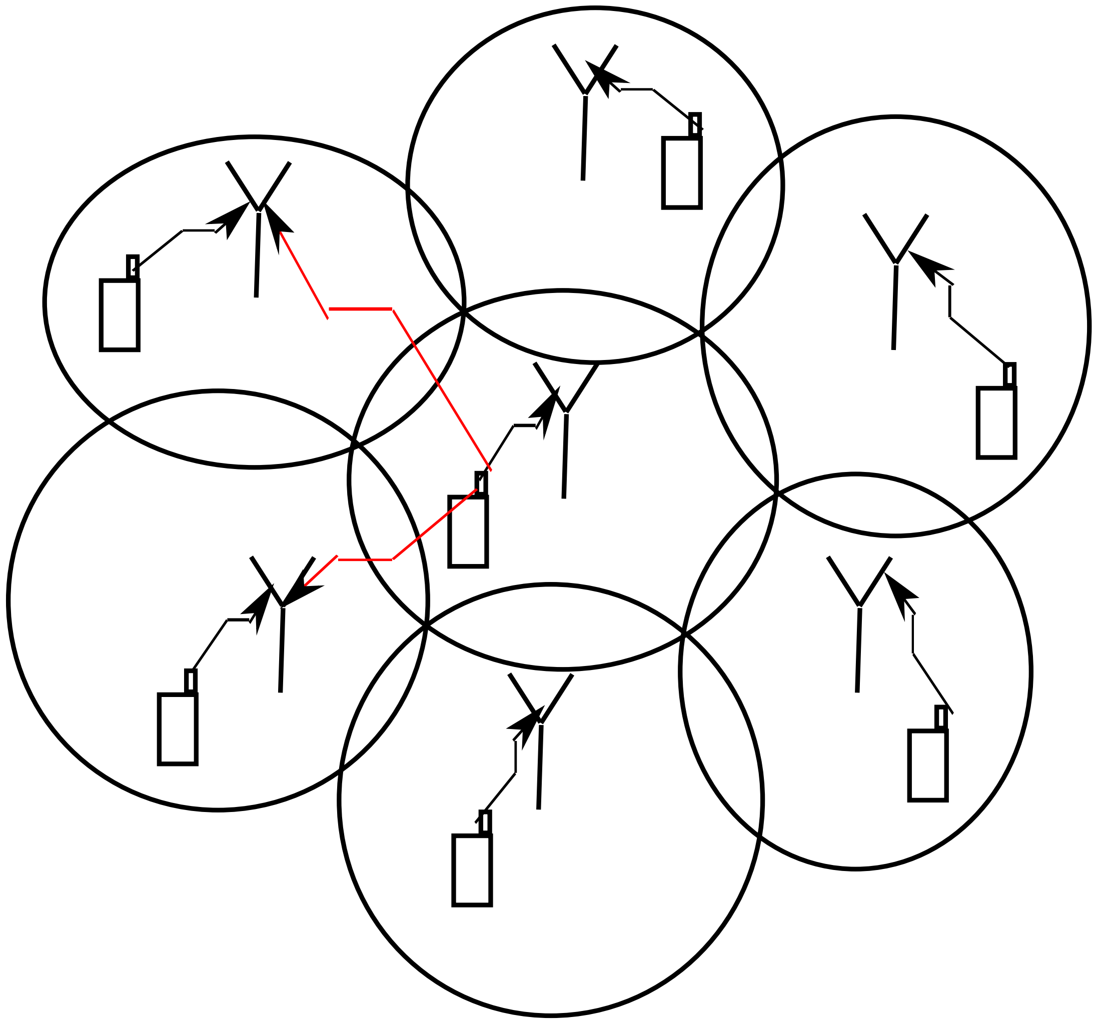
Cellular models



Cellular models



Introduction

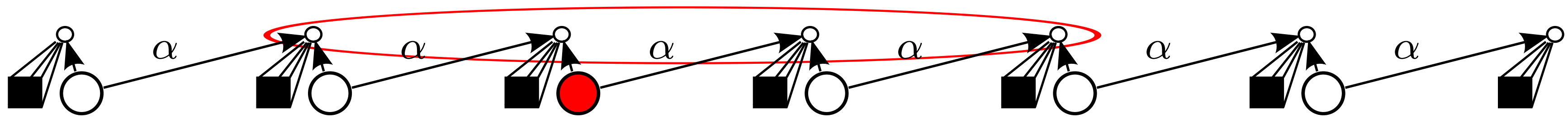
- For cellular models, *joint processing* of signals related to different users is a very appealing approach to enhance performances in either the uplink or downlink
- The Wyner model (uplink) [Wyner-94]:
  * Short range inter-cell interference
  * All the cell sites are linked by an ideal backhaul
  * Joint processing of all the received signals
- We assume that the the processing of the message sent by a given user is based only on the signals received in the cluster composed of the neighboring antennas:
  * Clustered local decoding
- We assume that every user knows the message that has to be sent by its neighbors:
  * Cognitive users

Previous work

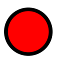
- [Wyner ’94] Global decoding
- [Shamai-Wyner ’97] Single-cell decoding
- Similar settings: [Shamai-Simeone-Somekh-Sanderovich-Zaidel-Poor ’08], [Somekh-Zaidel-Shamai ’07], [Atkas-Evans-Hanly ’04], [Shental-Weiss-Shental-Weiss ’04], [Shental-Weiss-Shental-Weiss ’07]
- Cognitive networks: [Lapidoth-Shamai-Wigger ISIT07] and [Lapidoth-Shamai-Wigger ITW07]

Clustered local decoding

General System Model



i_ℓ=1 and i_r=2



Transmitter 0

Receiver 0

- Independent transmitters on a line
- Antennas are set in the middle of every edge
- Every transmitter is received by the 2 adjacent antennas
- Attenuation parameter $0 < \alpha \leq 1$
- Decoding of the message sent by a transmitter using the signals received at the $i_\ell + i_r + 1$ adjacent antennas.
  - * $i_\ell + 1$ on the left
  - * i_r on the right
- The signals sum-up at the antenna along with AWGN (SNR=P)
- Focus on the per-user achievable rate $R(P)$

Non asymptotic results

- For a cluster of size 2:
  - * Analytical lower bounds are derived using Han-Kobayashi rate splitting
  - * Analytical upper bounds are derived using genie aided-bounds
- Results are derived for 2-dimensional models
- We focus on the high-SNR results: the high-SNR regime is characterized through the capacity approximation:

$$R(P) \underset{P \gg 1}{\approx} \mathcal{S}_\infty \log(P),$$

where $\mathcal{S}_\infty$ is the multiplexing gain or pre-log.

Multiplexing gain

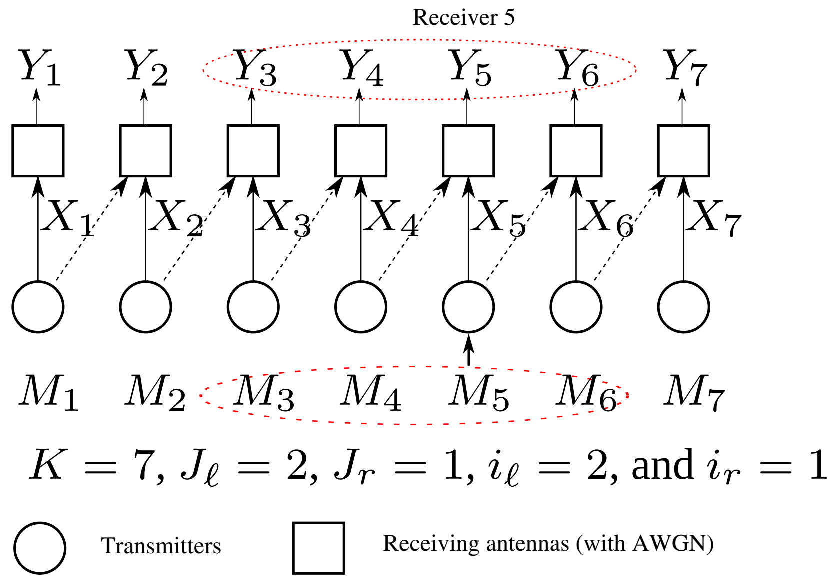
Theorem 1 *The multiplexing gain is*

$$\frac{i_\ell + i_r + 1}{i_\ell + i_r + 2}$$

- Lower bound:
  * silence every $(i_\ell + i_r + 2)$-th transmitter, the others can broadcast with multiplexing gain 1
  * first decode the messages at one edge then proceed by interference cancellation
- Upper bound: genie aided bound

Clustered local decoding and cognitive users

General System Model



- K single-antenna transmitters on a line
- K single-antenna receivers
- Every transmitter is received at its corresponding antenna and the one on its right
- Attenuation parameter $0 < \alpha \leq 1$
- The signals sum-up at the antenna along with AWGN (SNR=P)
- Transmitters have access to the messages of J_ℓ neighbors on the left and J_r neighbors on the right
- Receivers have access to the antennas of i_ℓ neighbors on the left and i_r neighbors on the right

Multiplexing gain

Theorem 2

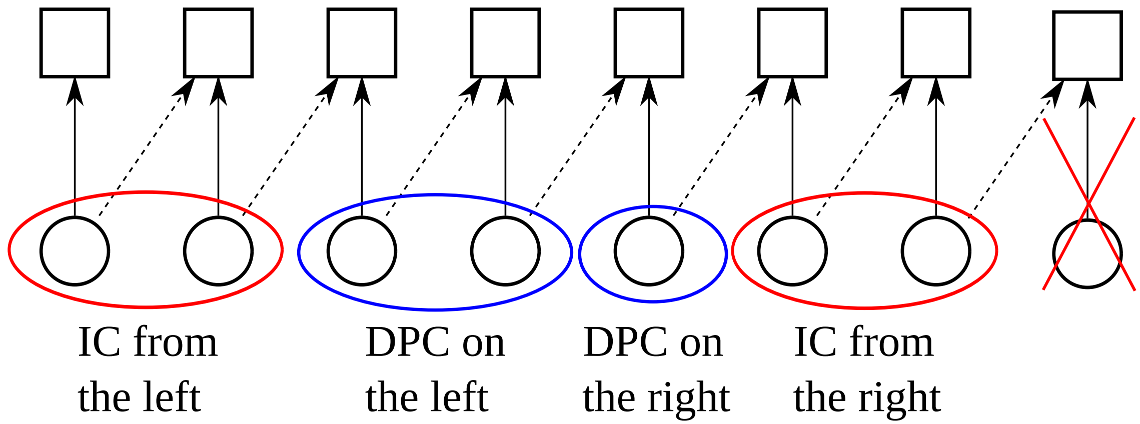
- *The asymptotic per-user multiplexing gain is*

$$\mathcal{S}_\infty = \frac{J_\ell + J_r + i_\ell + i_r + 1}{J_\ell + J_r + i_\ell + i_r + 2}$$

- Duality between information at the transmitter and information at the receiver
- By taking $J_\ell = J_r = 0$, we get our former result. By taking $J_r = i_\ell = i_r = 0$, we get [Lapidoth-Shamai-Wigger ISIT07].

Proof sketch of Theorem 2 - Lower bound

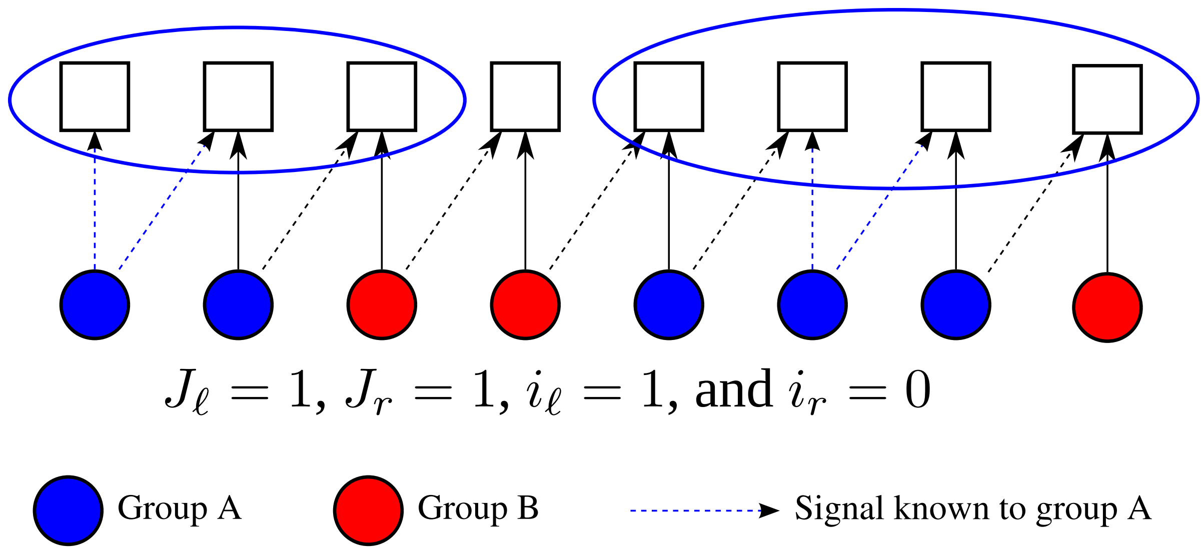
Case $i_r \neq 0$



$J_\ell = 2$, $J_r = 1$, $i_\ell = 1$, and $i_r = 2$

- Silence every $J_\ell + J_r + i_\ell + i_r + 2$-th transmitter and split the networks into independent sub-networks.
- The first $i_\ell + 1$ transmitters use interference cancellation from the left.
- The next J_ℓ transmitters use DPC to cancel the interference from the left.
- The next J_r transmitters use DPC to cancel the interference at the antenna on their right (to which their receiver has access).
- The next i_r transmitters use interference cancellation from the right.

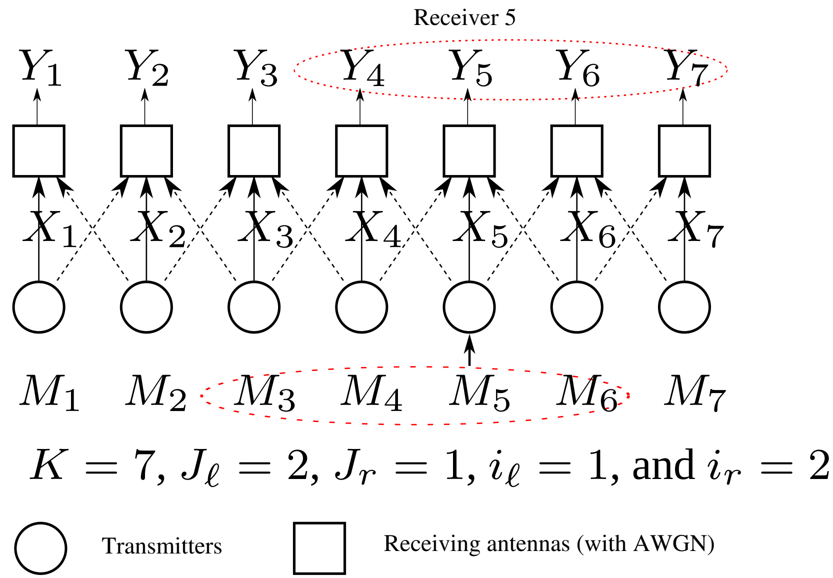
Proof sketch of Theorem 2 - Upper bound



$J_\ell = 1$, $J_r = 1$, $i_\ell = 1$, and $i_r = 0$

- Group the receivers in two groups and allow them to cooperate.
- Group A can reconstruct some of the broadcasted signals.
- Give to group A them extra information on the noise.
- With the extra information, the receivers of group A can reconstruct the received signal at all the antennas.
- The receivers of group A can decode all the messages therefore the multiplexing gain is bounded by the number of antennas to which they have access.
- The extra information on the noise does not change the multiplexing gain.

The full Wyner model



- WORK IN PROGRESS
- Every transmitter is received at its corresponding antenna and the ones on its right and left
- Results for $i_\ell + J_\ell = i_r + J_r$.
- Denote by $H_p(\alpha)$ the following matrix of size $p \times p$

$$\begin{pmatrix} 1 & \alpha & 0 & 0 \\ \alpha & 1 & \ddots & 0 \\ 0 & \ddots & \ddots & \alpha \\ 0 & 0 & \alpha & 1 \end{pmatrix}.$$

The full Wyner model *(Cont'd)*

- If α is such that $H_{i_\ell+J_\ell+1}(\alpha)$ is invertible, then

$$\mathcal{S}_\infty = \frac{i_\ell + i_r + J_\ell + J_r + 2}{i_\ell + i_r + J_\ell + J_r + 4}.$$

- If α is such that $H_{i_\ell+J_\ell+1}(\alpha)$ is not invertible, $\mathcal{S}_\infty$ is upper-bounded by

$$\frac{i_\ell + i_r + J_\ell + J_r + 1}{i_\ell + i_r + J_\ell + J_r + 3}.$$

- For example, $i_\ell = i_r = 1$, $J_\ell = J_r = 0$.

$$\mathcal{S}_\infty = 2/3 \text{ if } \alpha \neq 1 \text{ and } \mathcal{S}_\infty = 3/5 \text{ if } \alpha = 1.$$

- Open questions:
  * Lower bound for $i_\ell + J_\ell \neq i_r + J_r$.
  * Power off-set when $\det(H_p(\alpha))$ goes to 0.

Proof sketch of the lower bound

- Special case $i_\ell + J_\ell = i_r + J_r = 2$. $\alpha \neq \sqrt{2}/2$ such that H_3 is invertible.
- Silence every forth user and consider a sub-network of 3 consecutive users.
- Several cases:
  * $i_\ell = i_r = 2$: every receiver sees the three antennas: it is a MAC.
  * $i_\ell = J_\ell = i_r = J_r = 1$: every transmitter sees the second message and the second receiver sees all the antennas: every transmitter sends the second message; it is a MIMO channel.
  * $J_\ell = J_r = 2$: every transmitter sees all the messages: it is a broadcast channel.
  * $i_\ell = J_\ell = 1$, $J_r = 2$: every transmitter sees messages 2 and 3; receiver 2 sees antennas 1 and 2: it is a broadcast channel, message 2 is sent with pre-log 2 and message 3 is sent with pre-log 1.
- In any case, we need H_3 invertible.

Concluding Remarks

- Clustered local decoding:
  * The impact of clustered local decoding was demonstrated here through a simple analytically tractable (Wyner-like) multi-cell model
  * The results show the influence of the cluster-size on the multiplexing gain.
  * In the one dimensional model, the high-SNR multiplexing gain increases with complexity
    - 1/2 with individual cell decoding
    - $i/(i+1)$ with clustered local decoding, where i is the size of the cluster
    - 1 with full cell-site cooperation
  * Clustered local decoding is appealing on the practical side as a compromise between performances on one hand and system requirements (like backhaul) and complexity on the other hand.

Concluding Remarks *(Cont'd)*

- Clustered local decoding and cognitive users:
  - * The impact of clustered local decoding and cognitive users was demonstrated here through a simple analytically tractable (Wyner-like) multi-cell model
  - * The model includes a simple one-dimensional "soft handoff" scenario and the full one-dimensional Wyner setting (work in progress)
  - * Upper-bounds on the pre-log by grouping receivers and giving extra information
  - * Lower-bounds on the pre-log by schemes using "elementary elements" of the multi-users information theory.
  - * Symmetry between knowledge at the transmitter and knowledge at the receiver. For infinite number of users, only the sum $i_\ell + J_\ell + i_r + J_r$ matters.

The End

Thank You